Name: Key Date:

SOLVING LINEAR EQUATIONS
COMMON CORE ALGEBRA II

We will learn many new equation solving techniques in Algebra II, but the most basic of all equations are those where the variable, say x, is only raised to the first power. These are known as **linear equations**. You need to have good fluency with solving these equations in order to be successful in the beginning portions of Algebra II. Let's start with some practice.

***Exercise* #1:** Solve each of the following linear equations for the value of x. (Get x by itself)

(a) $3x+5=26$

$-5 \quad -5$

$\Rightarrow \frac{3x}{3}=\frac{21}{3} \Rightarrow \boxed{x=7}$

(b) $8x-\cancel{7}=4\cancel{x}-5$

$-4x+\cancel{7} \quad -\cancel{4x} \quad +7$

$\frac{4x}{4}=\frac{2}{4} \Rightarrow \boxed{x=\frac{1}{2}}$

(c) $2\cdot\frac{x+8}{2}=-6\cdot 2$

$x+\cancel{8}=-12$

$-\cancel{8} \quad -8$

$\Rightarrow \boxed{x=-20}$

(d) $6(x+4)-2(x-1)=2x+20$

$6x+24-2x+2=2x+20$

$4x+\cancel{26}=2\cancel{x}+20$

$-2x \quad -\cancel{26} \quad -\cancel{2x} \quad -26$

$\frac{2x}{2}=\frac{-6}{2}$ $\boxed{x=-3}$

It is important to understand that each step in solving one of these equations can be justified by either using one of the properties of real numbers (from the last lesson) or a property of equality (such as the additive or multiplicative properties).

***Exercise* #2:** Justify each step in solving $2(x+7)+4x=44$ using either a property of real numbers (commutative, associative, or distributive) or a property of equality (additive or multiplicative).

$2(x+7)+4x=44$
$2x+14+4x=44$ Distributive
$2x+4x+14=44$ Commutative
$x(2+4)+14=44$ Distributive
$6x+14=44$
$6x+14-14=44-14$ Additive
$6x=30$
$\frac{6x}{6}=\frac{30}{6}$ Multiplicative
$x=5$

Strange things can sometimes happen when solving linear (and other) equations. Sometimes we get no solutions at all, in which case the equation is known as **inconsistent**. Other times, any value of x will solve the equation, in which case it is known as an **identity**.

***Exercise* #3:** Try to solve the following equation. State whether the equation is an **identity** or **inconsistent**. Explain.

$$6x-2(x+4)=3(x+2)+x-5$$

$6x-2x-8=3x+6+x-5$

$4x-8=4x+1$

$-4x \quad -4x$

$\Rightarrow -8\neq 1$

Because the remaing equation is false (-8 does not equal 1), the equation is inconsistent.

***Exercise* #4:** An identity is an equation that is true for all values of the substitution variable. Trying to solve them can lead to confusing situations. Consider the equation:

$$2x-6+x-1=3(x-3)+2$$

(a) Test the values of $x=5$ and $x=3$ in this equation. Show that they are both solutions.

① $2(5)-6+5-1=3(5-3)+2$

$10-2=3(2)+2$

$8=6+2$

$8=8$ ✓

② $2(3)-6+3-1=3(3-3)+2$

$6-6+3-1=3(0)+2$

$2=0+2$

$2=2$ ✓

(b) Attempt to solve the equation until you are sure this is an identity.

$2x-6+x-1=3(x-3)+2$

$3x-7=3x-9+2$

$3x-7=3x-7$ ✓

Same on both sides.

$\therefore$ identity

***Exercise* #5:** Which of the following equations are identities, which are inconsistent, and which are neither?

(a) $8x-2(x+3)=5(x-1)+x$ $\quad -6\neq -5$

$8x-2x-6=5x-5+x$

$6x-6=6x-5$ $\boxed{\text{inconsistent}}$

$-6x \quad -6x$

(b) $\dfrac{4x+2}{2}+8=2x+9$ $\quad \boxed{\text{identity}}$

$2x+1+8=2x+9$

$2x+9=2x+9$

(c) $2x+8-(x-7)=2(2x-3)$

$2x+8-x+7=4x-6$

$x+15=4x-6$

$-x+6 \quad -x+6$

$3x=21$

$\boxed{x=7}$

$\boxed{\text{neither}}$

(d) $2x+1+2(x-1)=\dfrac{16x-4}{4}$